SIMULATIONS OF HEAD-TAIL DINAMICS IN BUNCHES WITH SPACE CHARGE USING HEADTAIL AND PATRIC CODES

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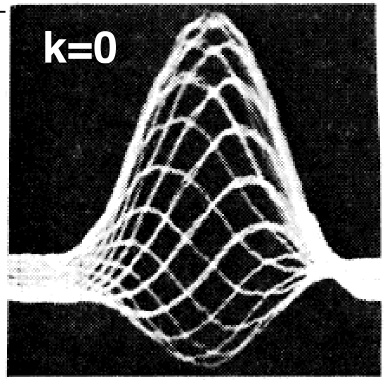
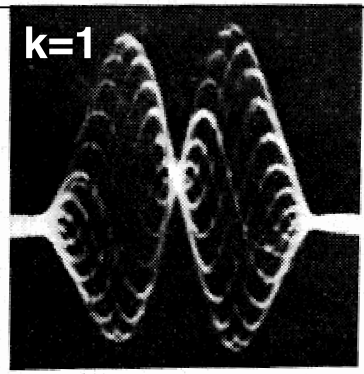
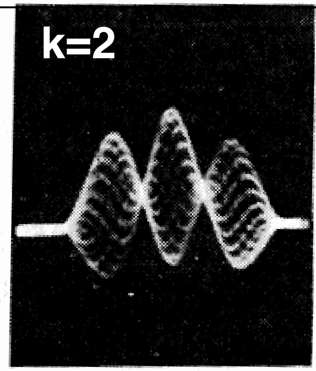
INTRODUCTION







Head-Tail Dynamics in SIS100 and PS changed by Space Charge (ΔQ_{SC} up to 0.25), not included by classical (Sacherer) theories

Previous (CERN-GSI meeting 19th February) comparisons left more questions then answers

Concluded that the modules:

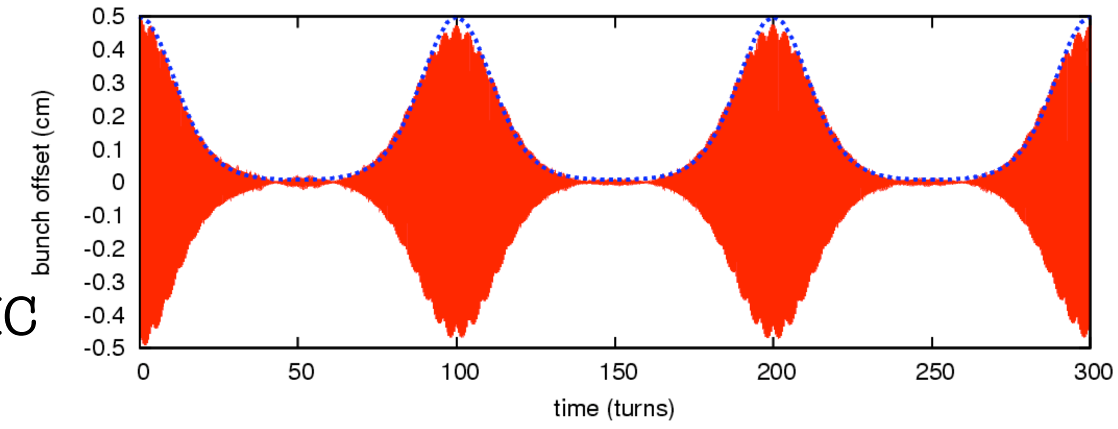
- Wake-field kicks
- Space-charge model

must be checked.

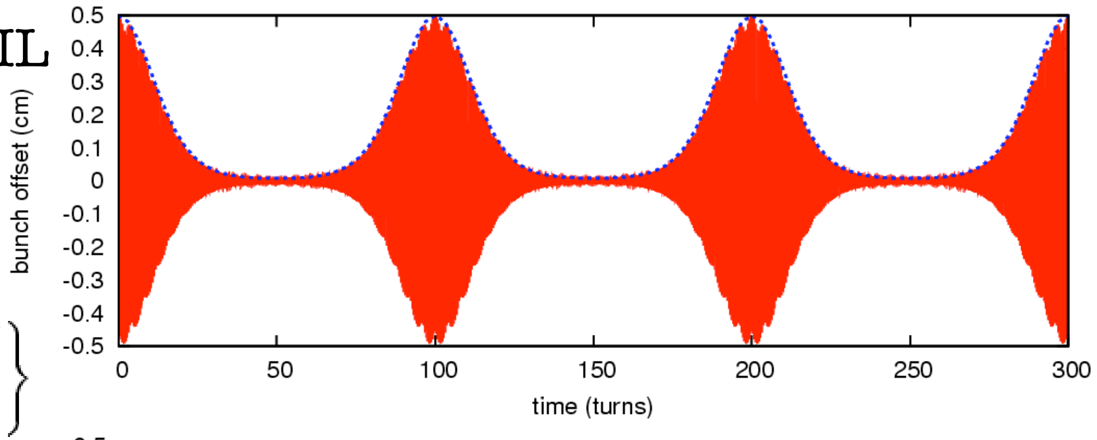
SIMPLE DECOHERENCE

First comparison:
Decoherence and recoherence
of a Gaussian bunch
Q_s=0.01

PATRIC



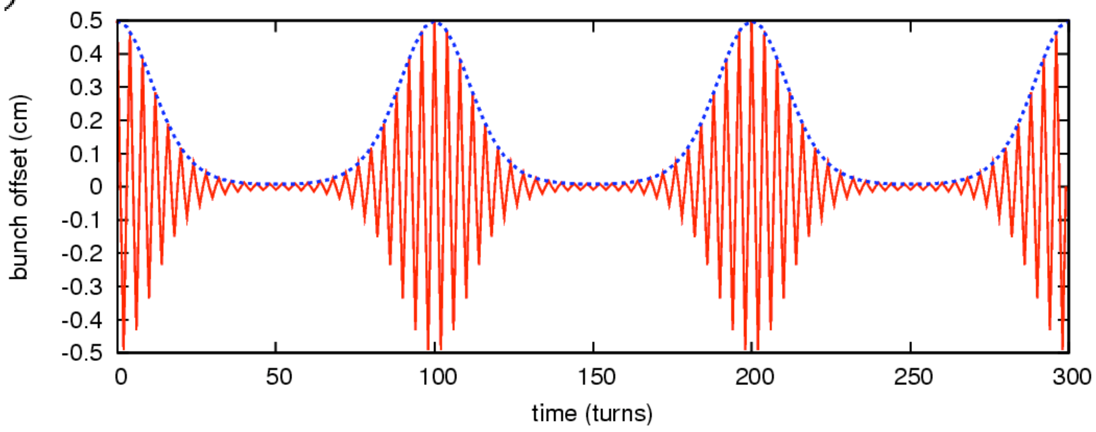
HEADTAIL



blue line is the theory:

$$\overline{a} = a_0 \exp\left\{-2\left[\frac{\delta p_{\mathrm{rms}}\xi Q_\beta}{Q_s}\sin(\pi Q_s N)\right]^2\right\}$$

HEADTAIL
1 kick per turn



good agreement

it is enough to kick once per turn

BUNCH DECOHERENCE

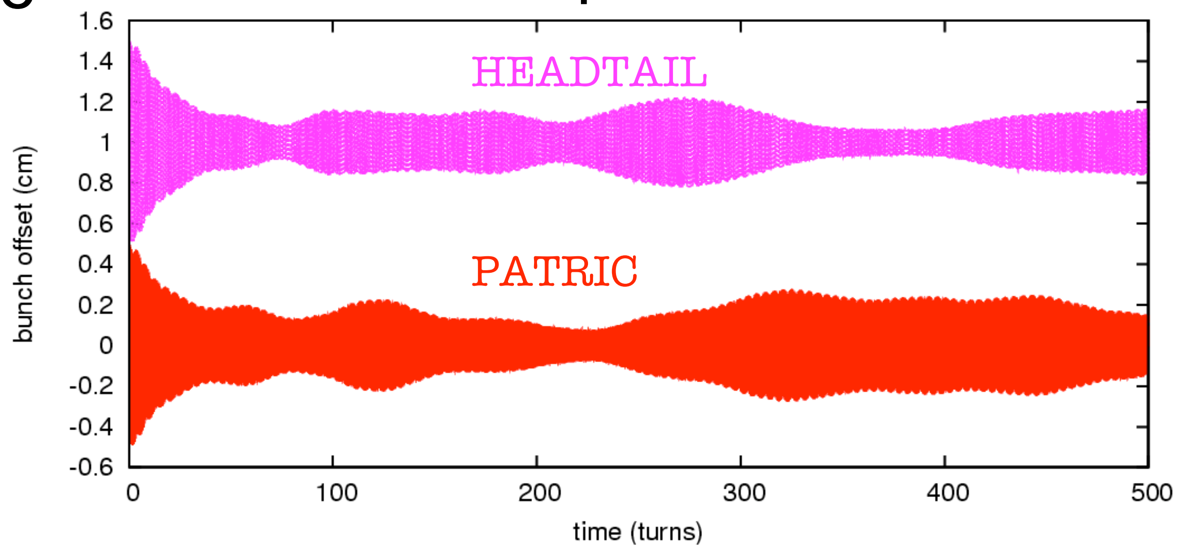
now include Space Charge

define

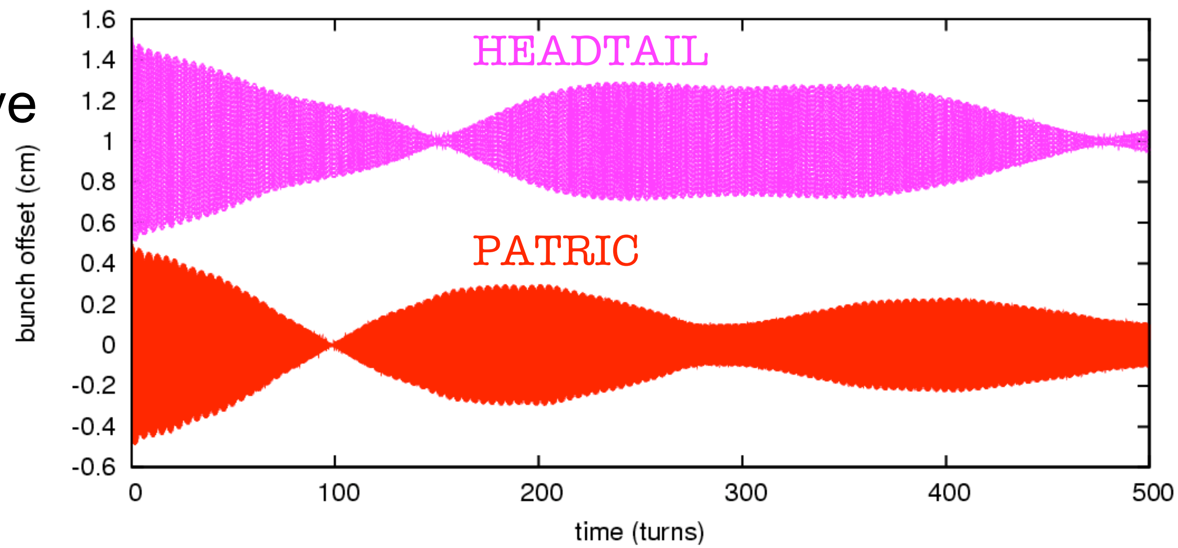
$$q = \frac{\Delta Q_{SC}}{Q_s}$$

Q_s=0.01

q=3



q=9



however, detailed and quantitive comparison is very difficult

- no theory
- very sensitive

AIR-BAG BUNCH

M.Blaskiewicz, PRSTAB **1**, 044201 (1998)

square-well potential
$$U(\tau) = \begin{cases} 0, & \text{if } 0 < \tau < \tau_b\,; \\ U_0, & \text{otherwise}\,. \end{cases}$$

longitudinal distribution
$$f(p) \propto \delta(p - p_0) + \delta(p + p_0)$$

$$\begin{aligned} X_+(\theta,\tau) &= e^{-iQ\theta} x_+(\theta,\tau) \\ X_-(\theta,\tau) &= e^{-iQ\theta} x_-(\theta,\tau) \\ \frac{\partial x_+}{\partial \theta} &\ll Qx_+ \end{aligned}$$

$$W(\tau) = W_0 \exp(-\alpha\tau)$$

$$\Downarrow$$

analytic solution

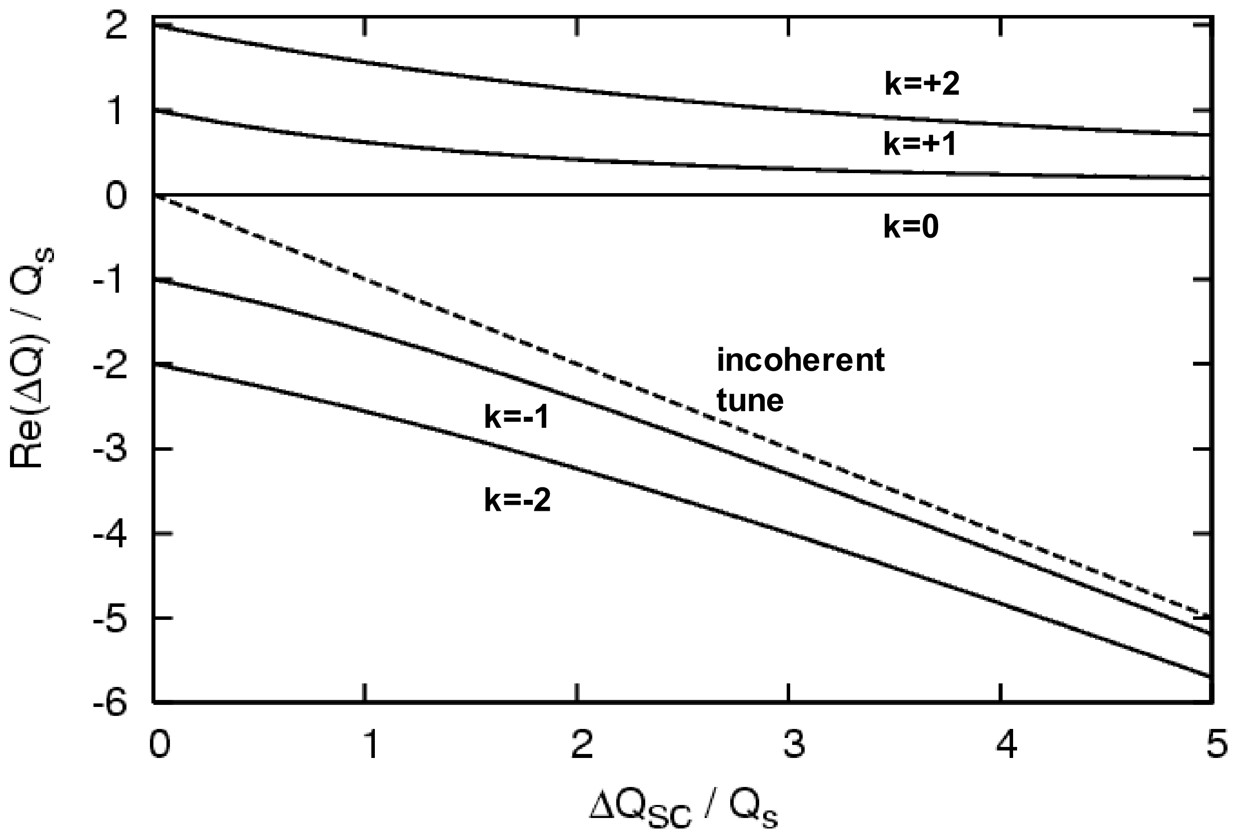
AIR-BAG BUNCH WITH SC

M.Blaskiewicz, PRSTAB **1**, 044201 (1998)

without wake

$$\Delta Q = -\frac{\Delta Q_{sc}}{2} \pm \sqrt{\frac{\Delta Q_{sc}^2}{4} + k^2 Q_s^2}$$

"+" for $k \geq 0$

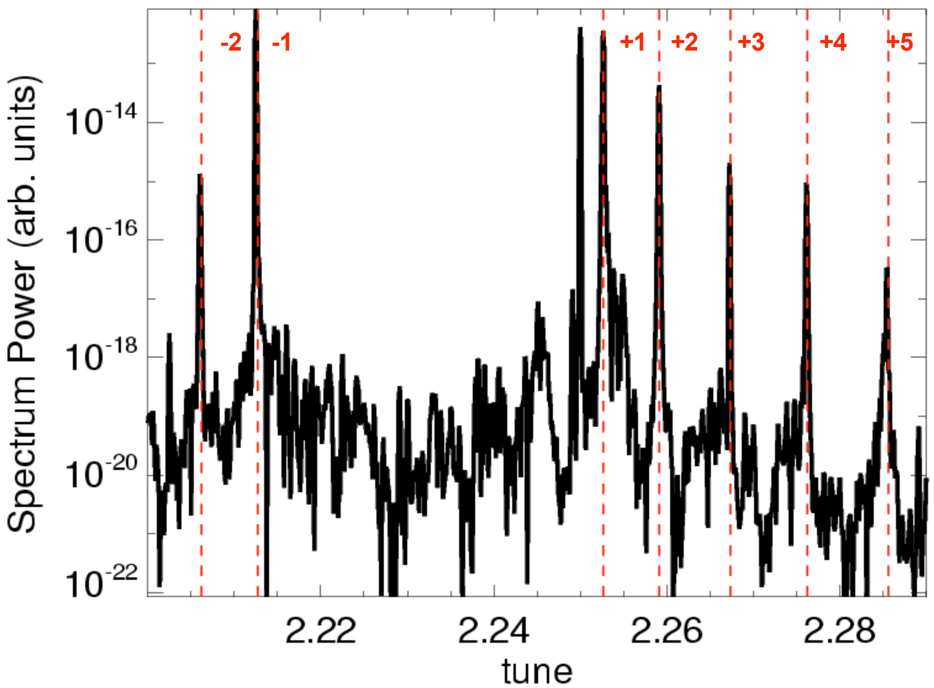


AIR-BAG BUNCH WITH SC

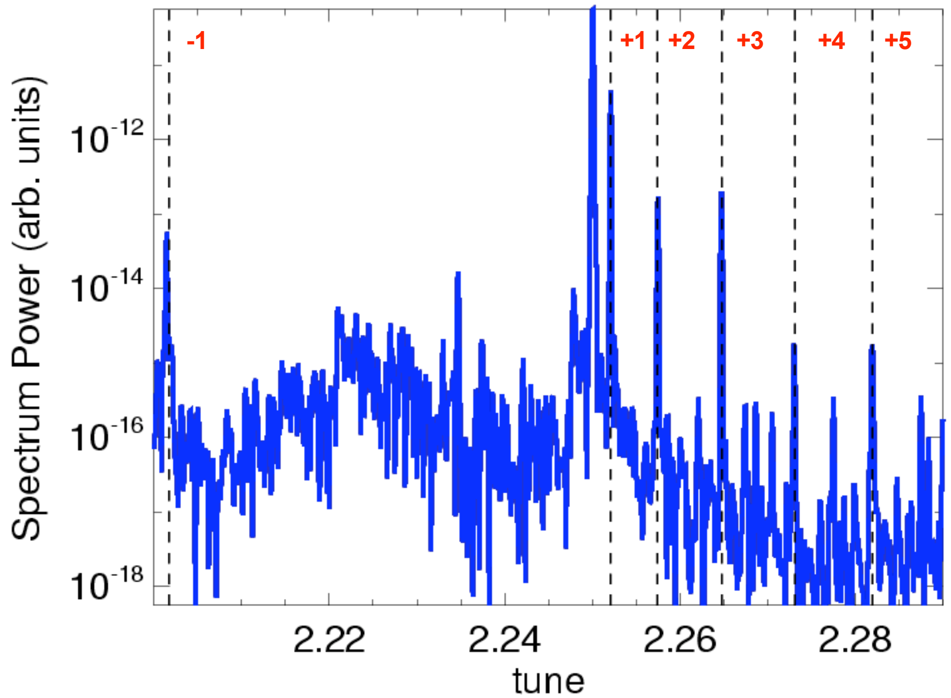
bunch spectrum (FFT from total bunch offset)

dash lines are the theory: $\Delta Q = -\frac{\Delta Q_{sc}}{2} \pm \sqrt{\frac{\Delta Q_{sc}^2}{4} + k^2 Q_s^2}$ "+" for $k \geq 0$

HEADTAIL
Q_s=0.01, q=3.0



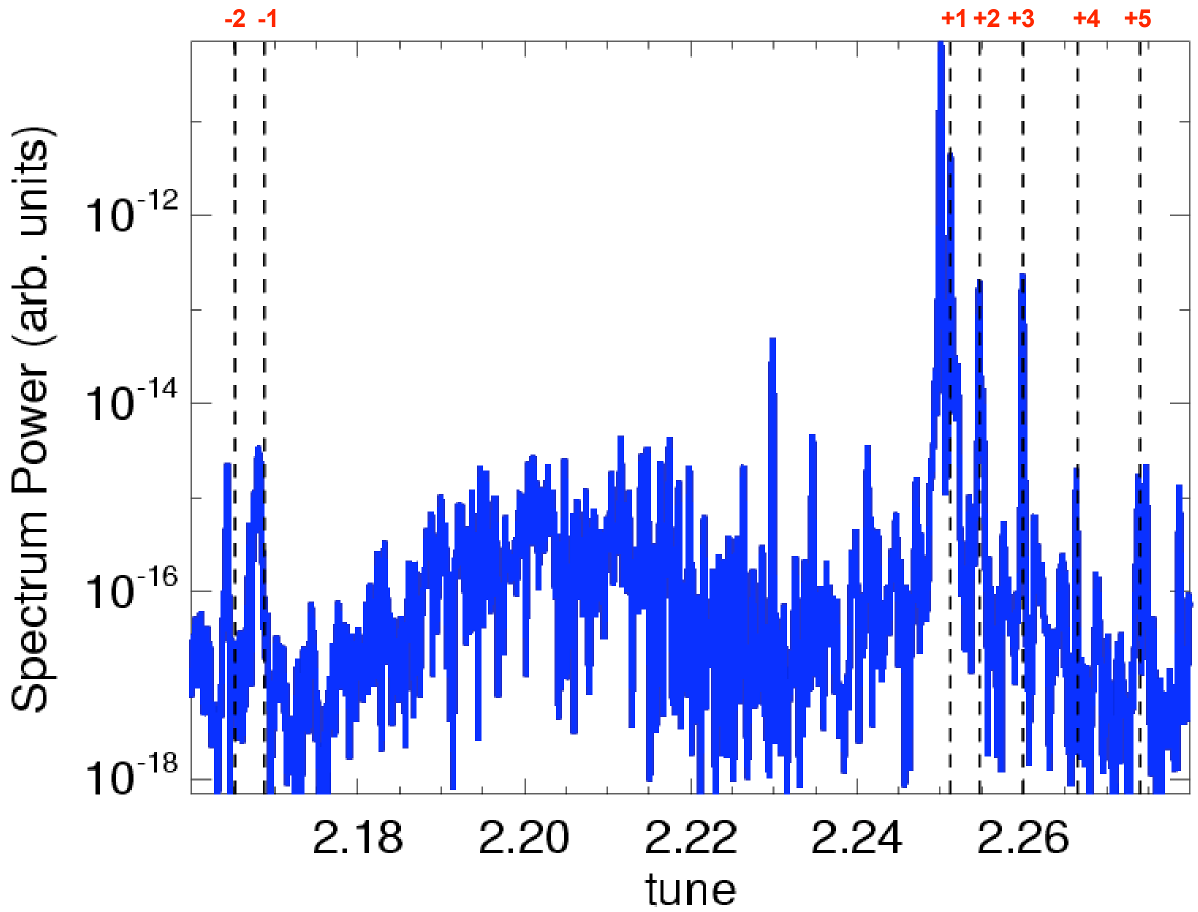
PATRIC
Q_s=0.01, q=4.62



good agreement achieved

AIR-BAG BUNCH WITH SC

even stronger space charge
PATRIC q=8.0



AIR-BAG BUNCH WITH SC

This space charge:
frozen electric field, moves with the slice center

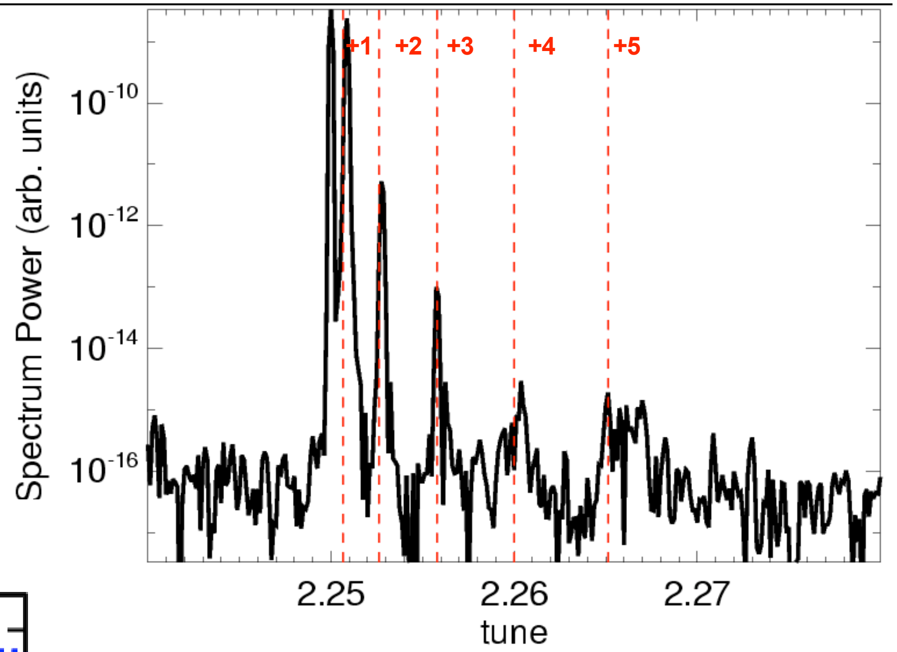
study with `HEADTAIL`: one kick is not enough!

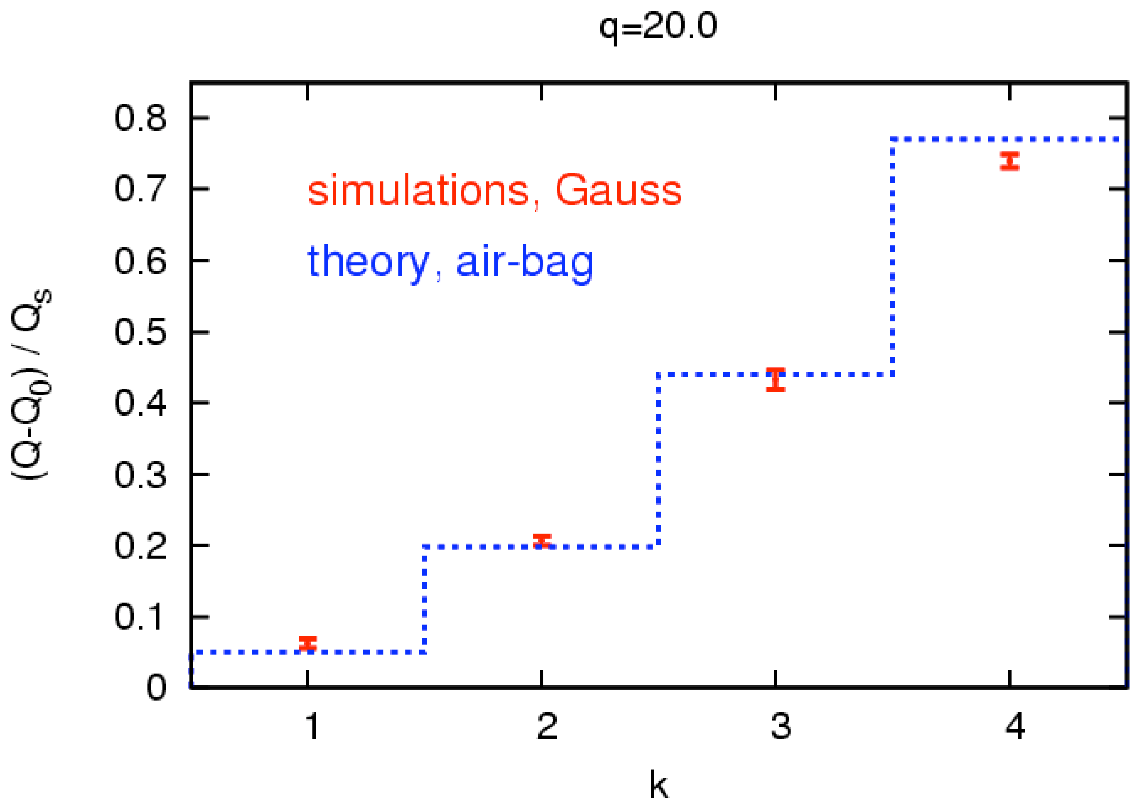
phase advance 1.4 not enough
phase advance **≈1.0** is enough
(tested with different bare tunes Q_0)

This means:
Q_0=6.2: 40 kicks
Q_0=18.7: 120 kicks

GAUSS BUNCH WITH SC

HEADTAIL simulations
for the Gaussian bunch (tr. KV)
Spectrum example for q=15





Bunch form seems to be not very important!

Tune shifts are well predicted by the air-bag theory, especially for strong SC

UNSTABLE AIR-BAG BUNCH

M.Blaskiewicz, PRSTAB **1**, 044201 (1998)

$$W(\tau) = W_0 \exp(-\alpha\tau)$$

$$\alpha\tau_b \gg 1$$

for k=0

$$\Delta Q = \Delta Q_0(\alpha/\zeta + i) \qquad \left(\zeta = \frac{\xi Q}{\eta}\right)$$

$$\Delta Q_0 = -\frac{\kappa\zeta}{\alpha^2} W_0$$

and for $k \neq 0$

$$\Delta Q = -\Delta Q_{sc} + \frac{\Delta Q_0(\alpha/\zeta + i) + \Delta Q_{sc} \pm \sqrt{[\Delta Q_0(\alpha/\zeta + i) + \Delta Q_{sc}]^2 + 4k^2 Q_s^2 [1 - (\Delta Q_0 \pi / 2\zeta Q_s \tau_b)^2]}}{2\,[1 - (\Delta Q_0 \pi / 2\zeta Q_s \tau_b)^2]}$$

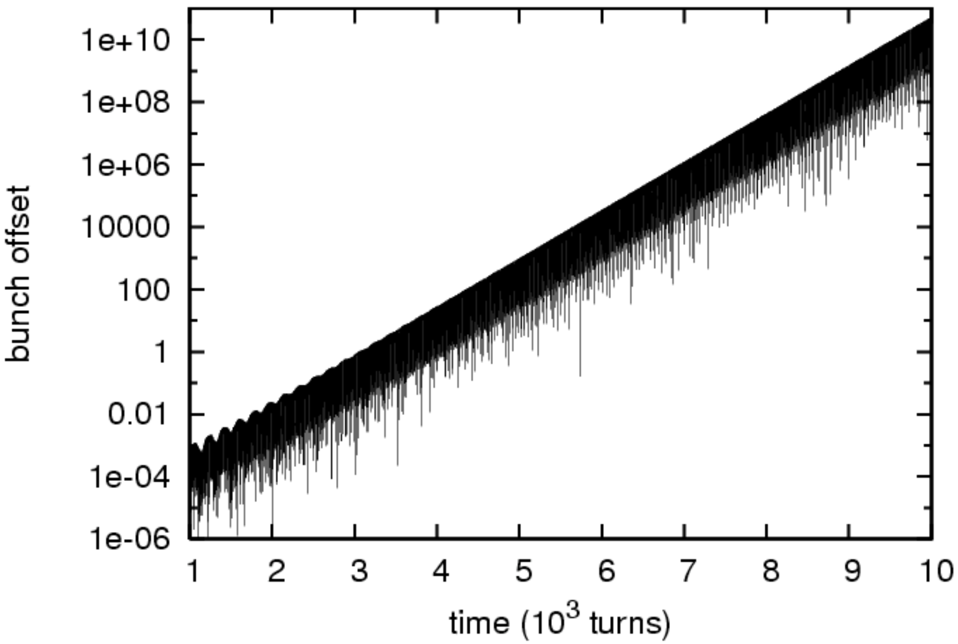
UNSTABLE AIR-BAG BUNCH

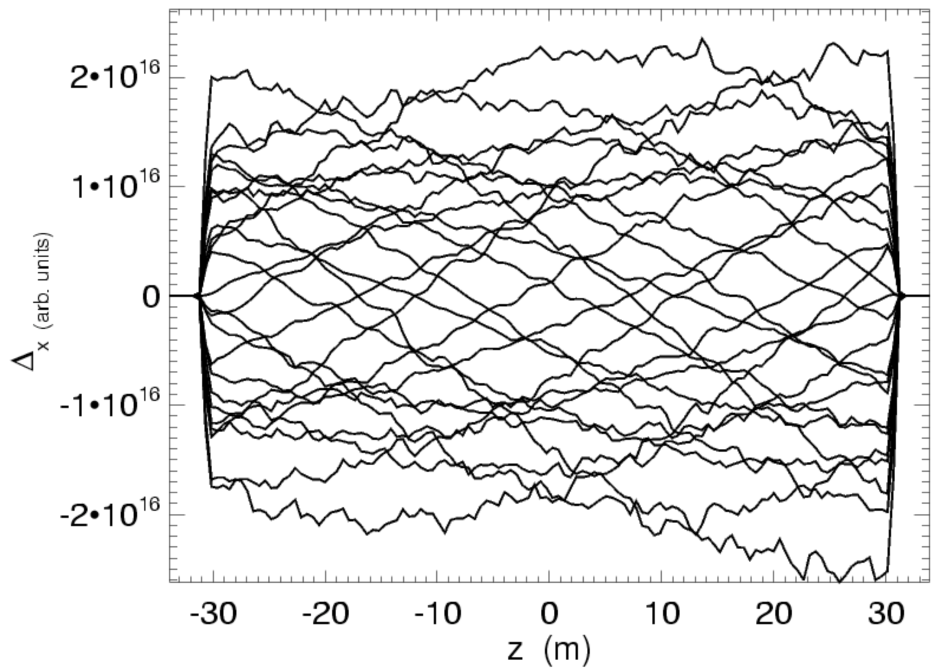
example Q_s=0.01 ΔQ_0=6.e-4

typical k=0 mode

PATRIC simulations

ΔQ_0=2.25, ξ=+2, without space charge

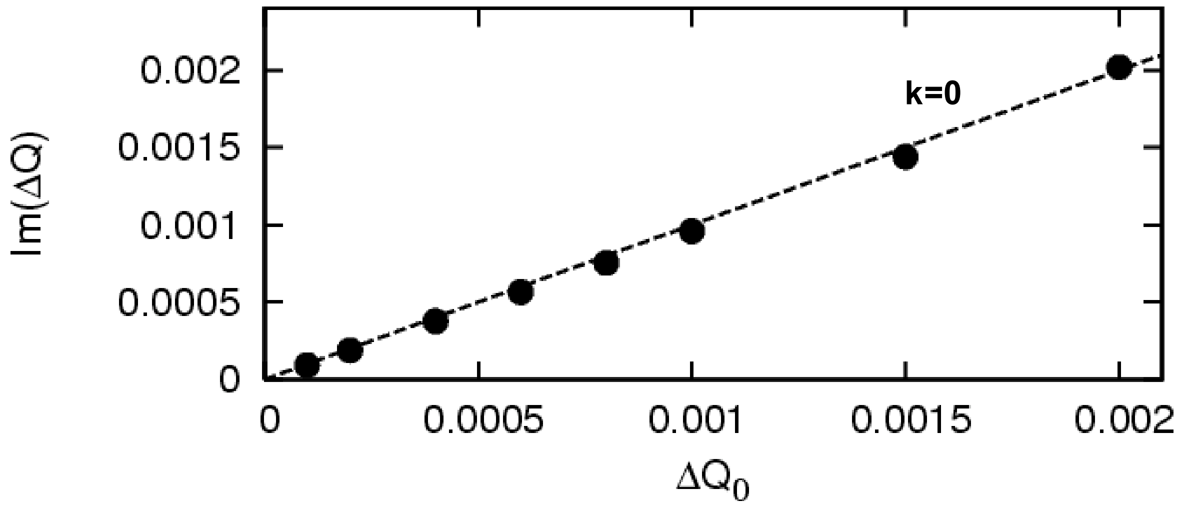




with this wake potential we always obtain the k=0 mode

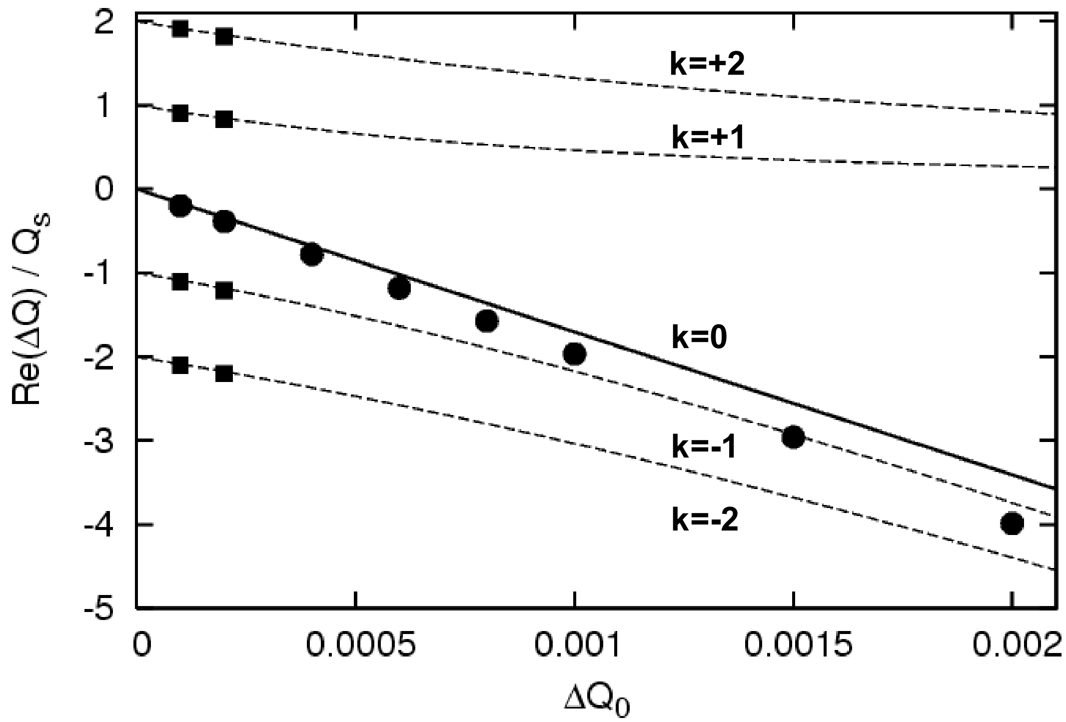
UNSTABLE AIR-BAG BUNCH

increasing wake
PATRIC without SC



for k=0

$$\Delta Q = \Delta Q_0(\alpha/\zeta + i)$$

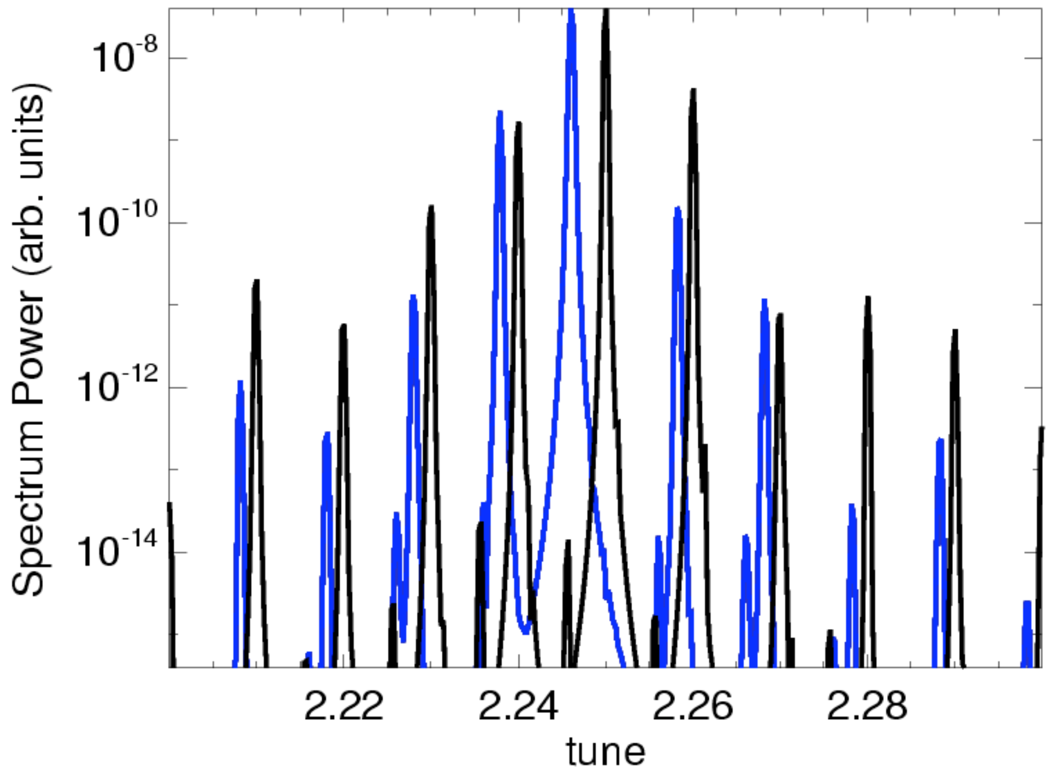


lines: air-bag theory
symbols: simulations

UNSTABLE AIR-BAG BUNCH

spectrum example (ΔQ_0=2e-4)

Q_0=2.25
black: without Wake
blue: with Wake



UNSTABLE AIR-BAG BUNCH

Include space charge

Increasing wake by constant ΔQ_{SC}

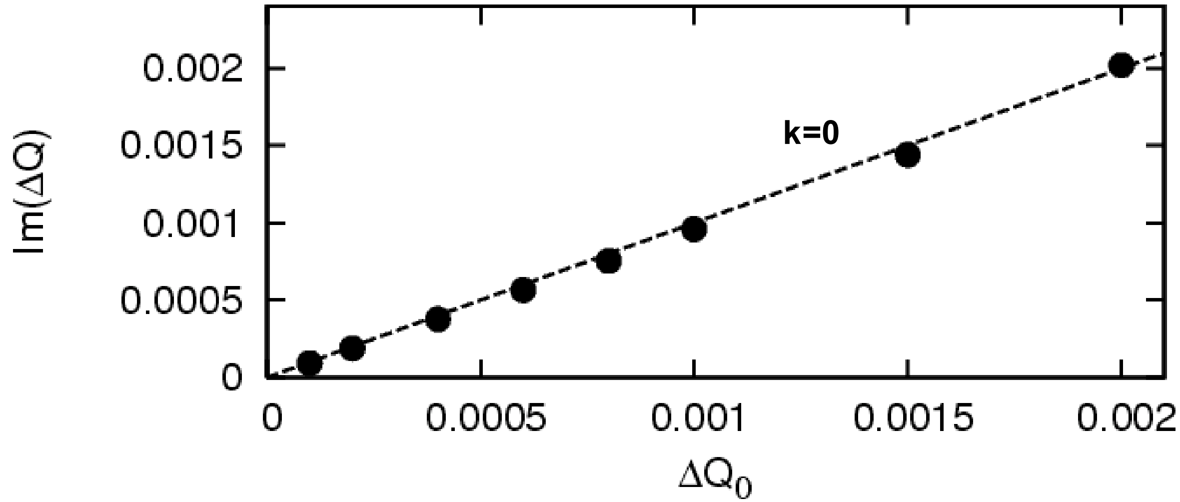
PATRIC with SC q=3

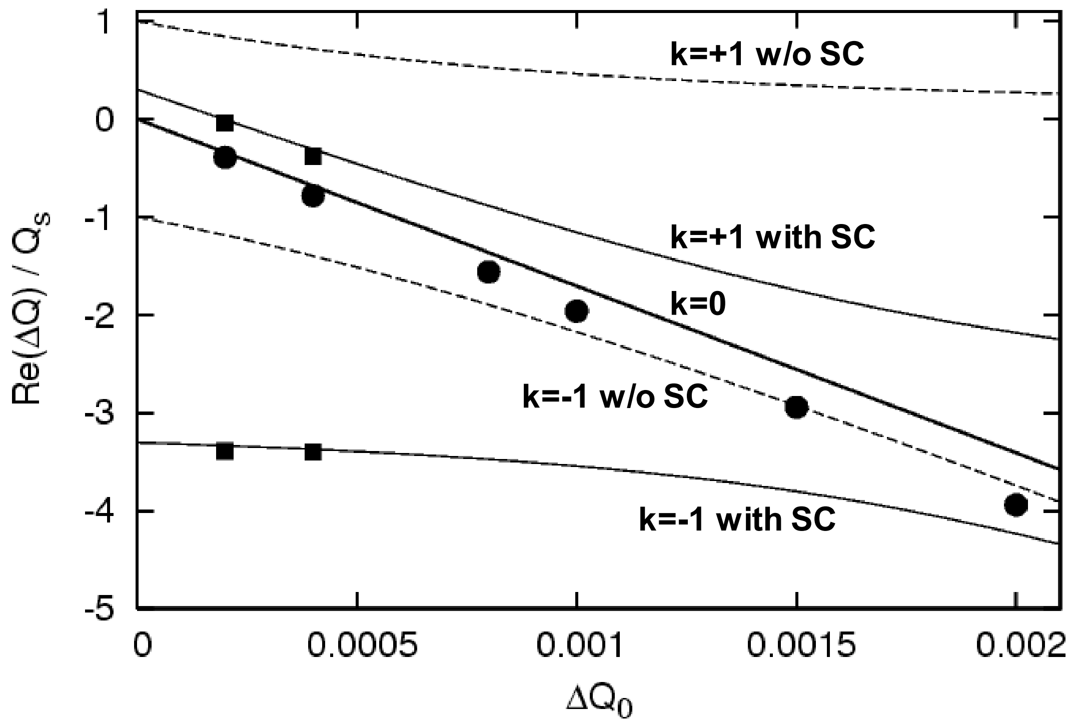
for k=0

$$\Delta Q = \Delta Q_0(\alpha/\zeta + i)$$

k=0 is not affected by SC

lines: air-bag theory
symbols: simulations



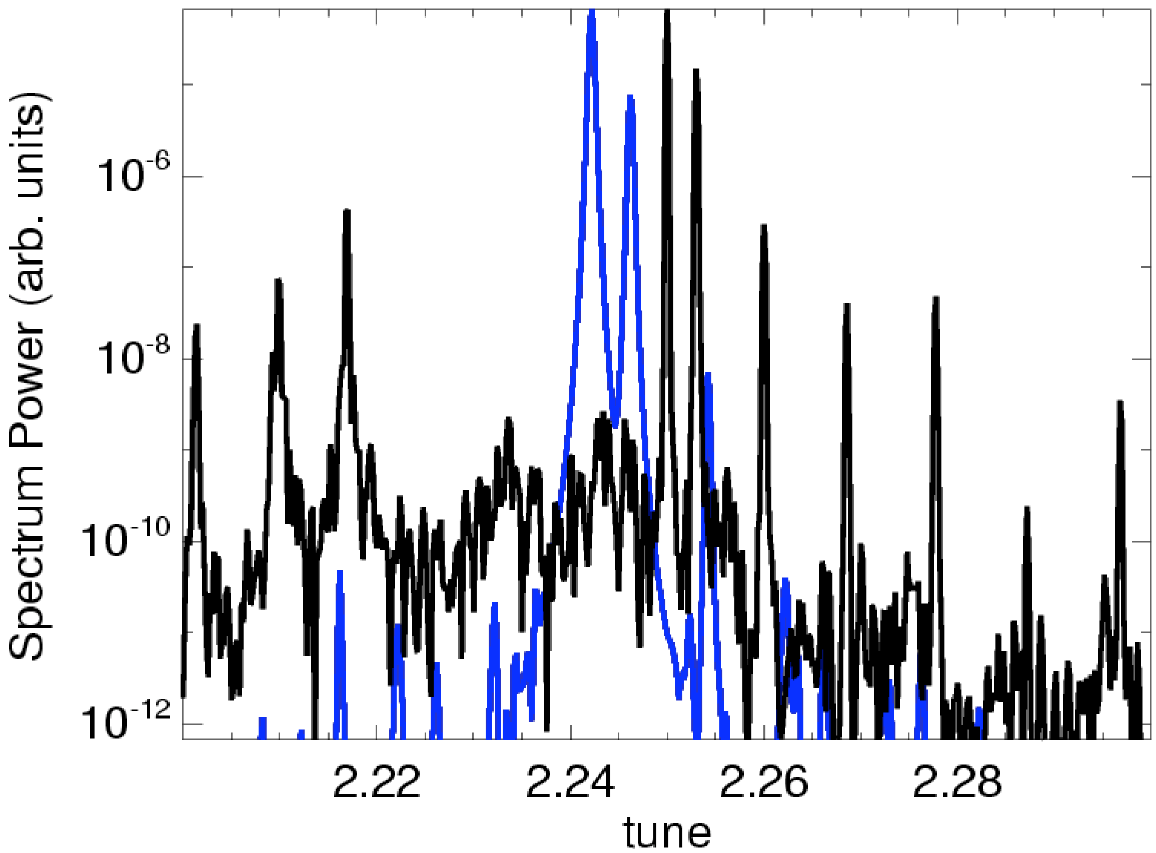


UNSTABLE AIR-BAG BUNCH

including space charge into instability simulations

spectrum example (ΔQ_0=4e-4)
PATRIC with SC q=3

black: without Wake
blue: with Wake

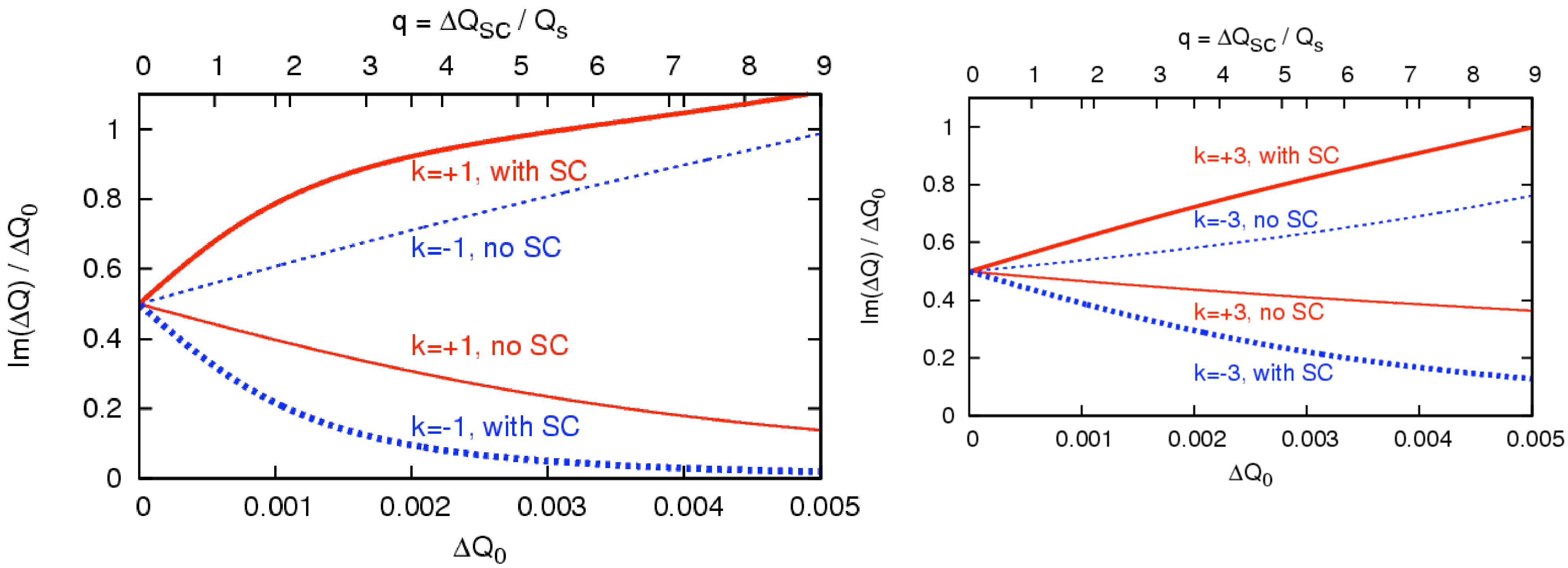


UNSTABLE AIR-BAG BUNCH

increasing intensity:
space-charge tune shift ΔQ_{SC} grows prop. to the wake
$\alpha\tau_b$=10

for k=0 $\quad \Delta Q = \Delta Q_0(\alpha/\zeta + i)\,,\qquad$ and for $k\neq 0$

$$\Delta Q = -\Delta Q_{sc} + \frac{\Delta Q_0(\alpha/\zeta + i) + \Delta Q_{sc} \pm \sqrt{[\Delta Q_0(\alpha/\zeta + i) + \Delta Q_{sc}]^2 + 4k^2 Q_s^2[1 - (\Delta Q_0 \pi / 2\zeta Q_s \tau_b)^2]}}{2\,[1 - (\Delta Q_0 \pi / 2\zeta Q_s \tau_b)^2]}$$



UNSTABLE AIR-BAG BUNCH

study with HEADTAIL:
in simulations with the wake (but w/o SC)
1 kick per turn is enough

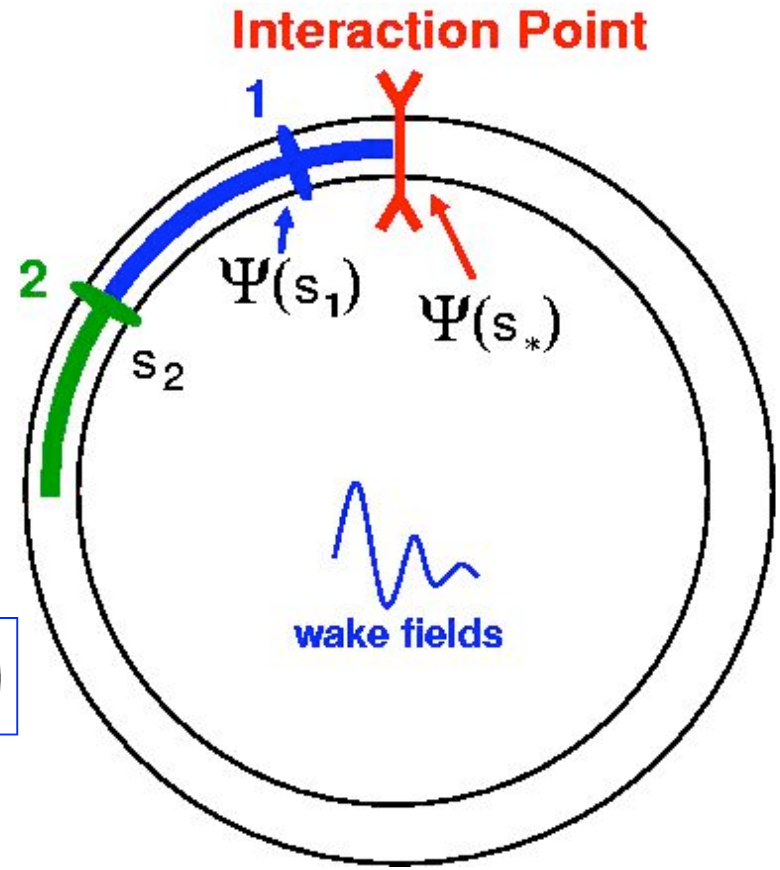
simulations with space charge for
the Gaussian bunch are more robust
than for the airbag bunch

WAKE-FIELD MODULE

Resistive wall (thick-wall) wake field

$$W_{\mathrm{RW}}(z) = -\frac{cL_{\mathrm{wall}}}{b^3}\sqrt{\frac{Z_0}{z\ \sigma_{\mathrm{wall}}}\left(\frac{\beta}{\pi}\right)^3}$$

$$\begin{aligned}\Delta p_y &= \int \Psi(s)\ W_{\mathrm{RW}}(z)\mathrm{d}z \\ \Psi &= k\ n_{\mathrm{p}}\overline{x} \\ z &= s_1 - s_2\end{aligned}$$



LONG BUNCHES
two approaches:
with / without phase advance:

$$\boxed{\Delta p_{12} = \Psi(s_1)\ W_{\mathrm{RW}}(z)}$$

$$\boxed{\Delta p_{12} = \Psi(s_*)\ W_{\mathrm{RW}}(z)}$$

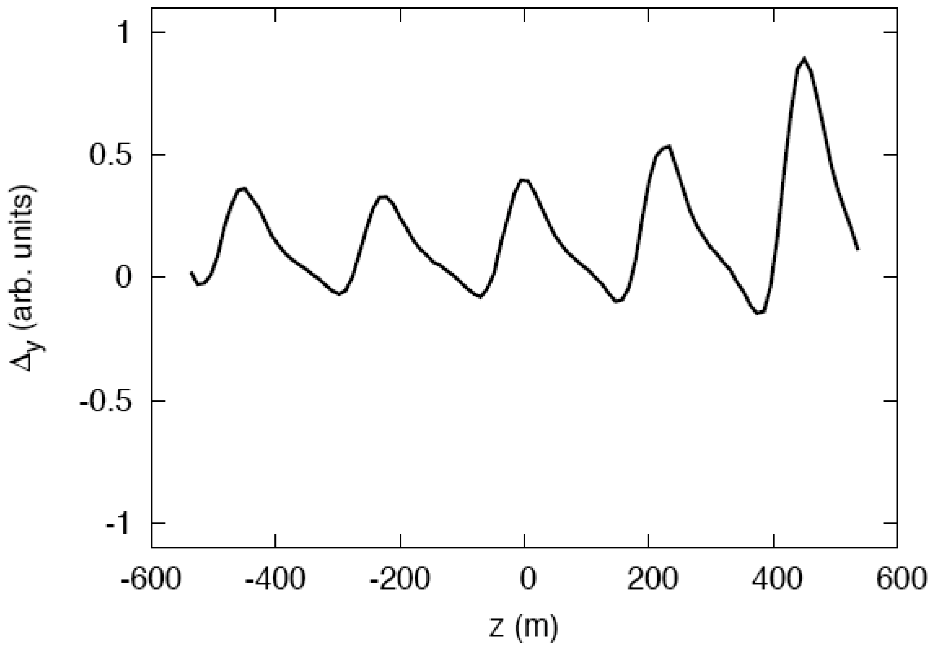
WAKE-FIELD MODULE

the standpoint: if something is made for long bunches,
it should work for **coasting beam**

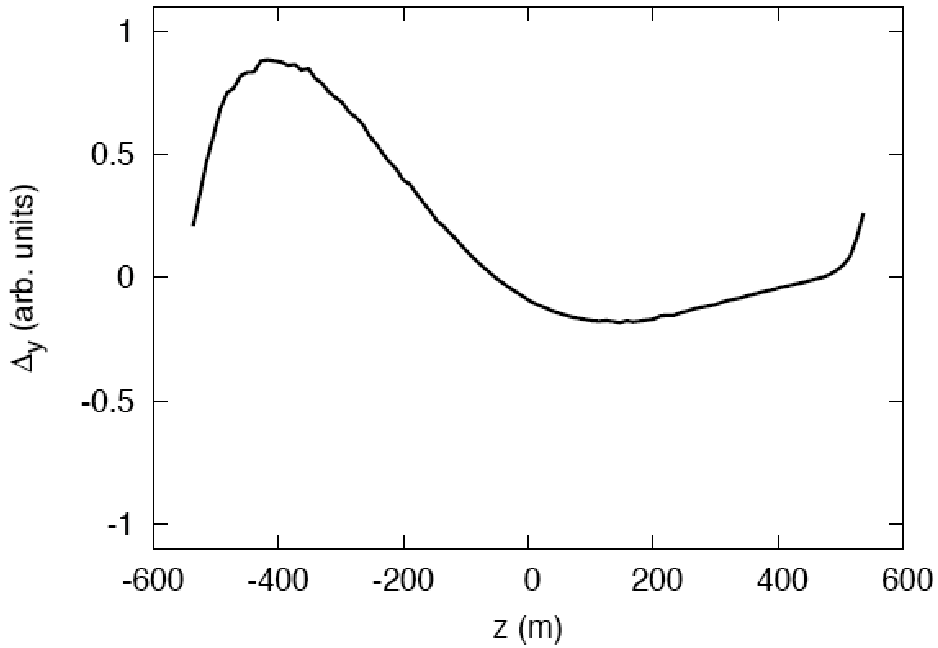
$f_{slow}=(n-Q)\ f_0$ $Q=Q_{int}+Q_{fr}$ (this example: 4.4=4+0.4)

RW at low frequencies $\Rightarrow$ $n=Q_{int}+1$

with phase advance



without phase advance



dipole signal of an unstable coasting beam

WAKE-FIELD MODULE

This approach: $W(\tau) = W_0 \exp(-\alpha\tau)$

$\alpha\tau_b \gg 1$

$$\frac{\partial x_+}{\partial\theta} \ll Qx_+$$

(have tried the "phase approach" $\Rightarrow$ wrong ΔQ)

or for short bunches and high-freq. wake $L_{bunch} \gg L_{wake}$

so $\Delta p = \int_0^\tau W(\tau - s)\Psi(s)\mathrm{d}s$ is correct,

but what is with:

- our solutions without $X(\theta,\tau) = e^{-iQ\theta}x(\theta,\tau)$
- resistive-wall impedance
- long bunches

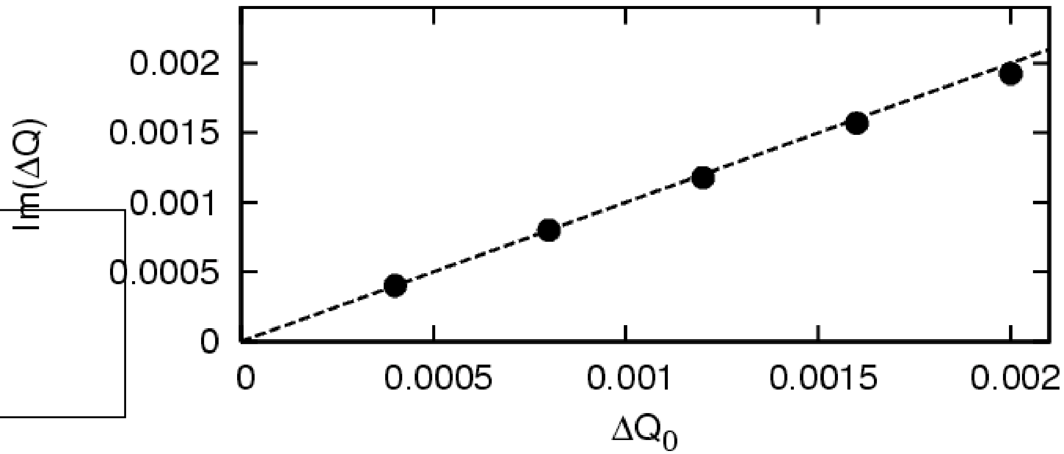
$\Rightarrow$ first-principles study necessary?

UNSTABLE GAUSS-BUNCH

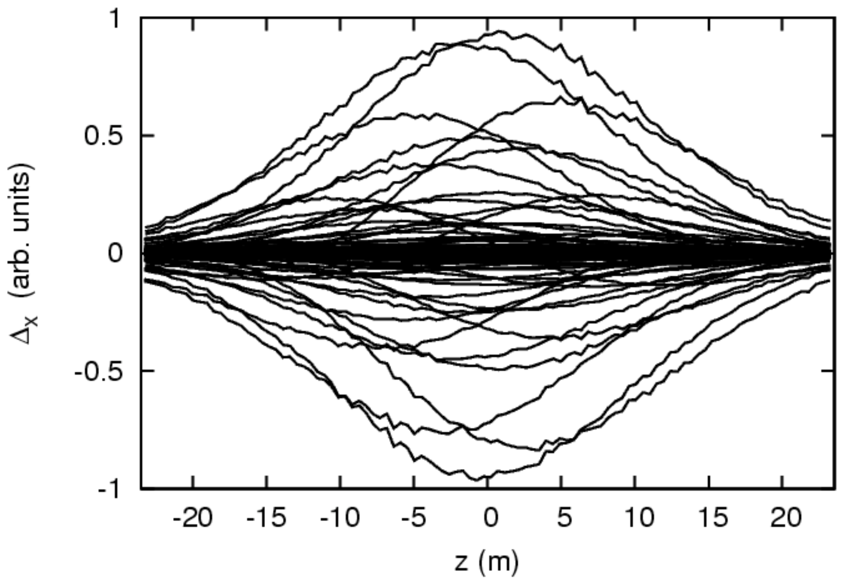
increasing wake

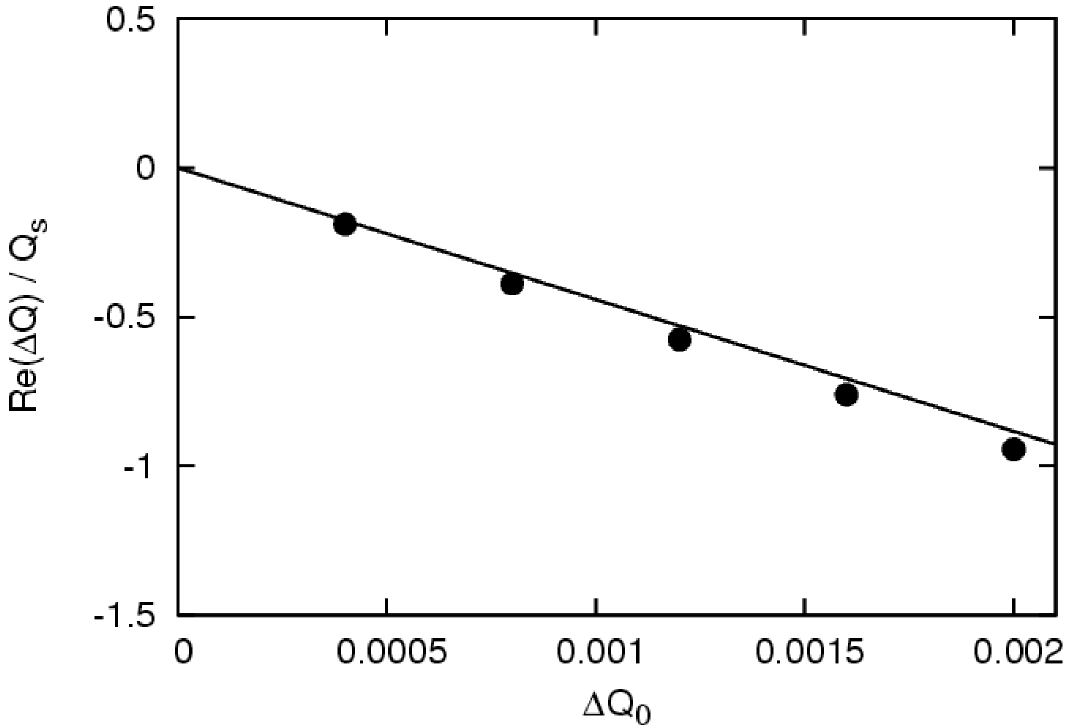
HEADTAIL with SC q=5

lines: air-bag theory
symbols: simulations
ΔQ_{simul} scaled by the factor 1/0.53



k=0 is not affected by SC

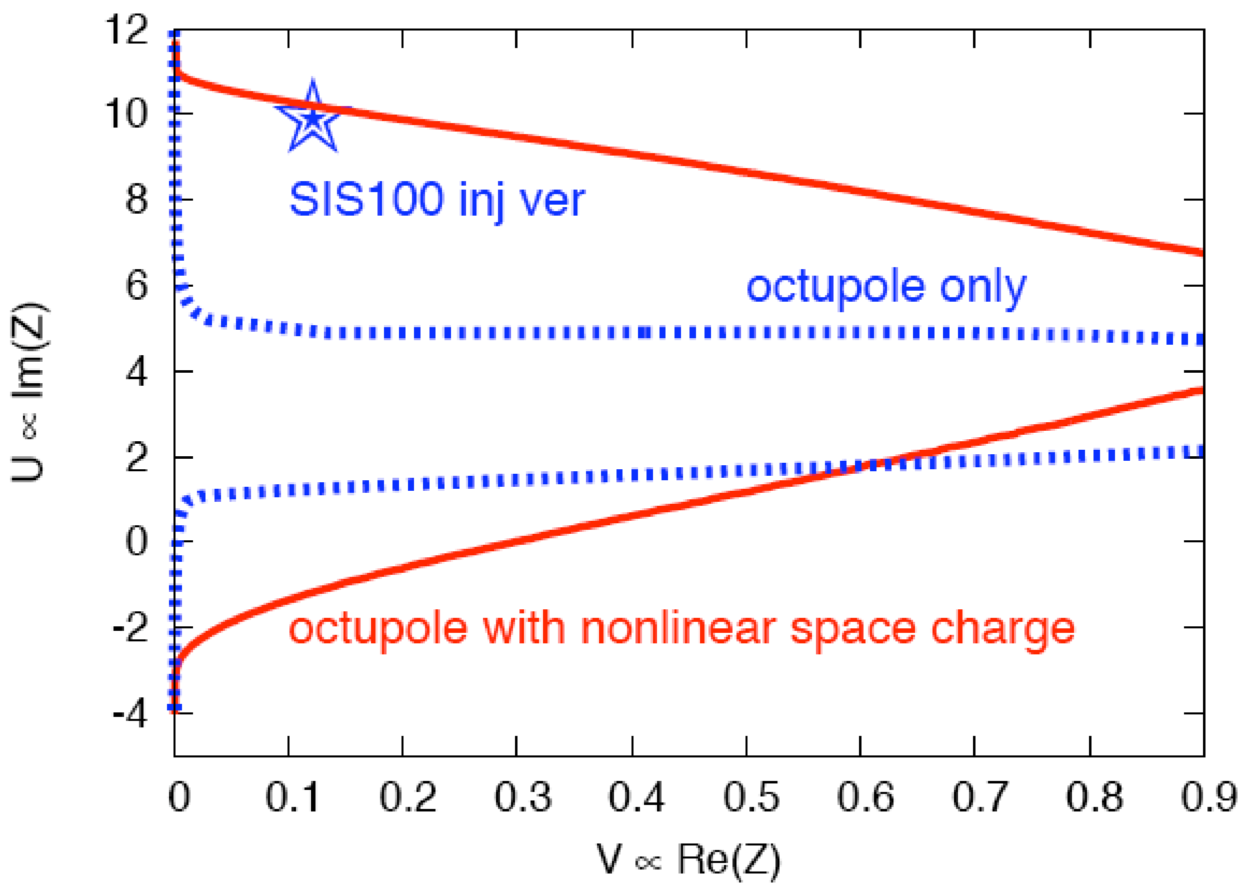




OCTUPOLES

Predictions for coasting beam, SIS100, injection energy, vertical RW

SIS100 Magnets:
12 octupoles,
length 75 cm,
max. 2000 T/m^3



damping of head-tail modes?
demonstration?
PS experiments?

SUMMARY

Space Charge and Wake Field Modules:
good agreement achieved in detailed
comparisons of the code results with the air-bag theory

Simulations with the Gaussian bunch
(both space charge and head-tail instabilities) show that
the bunch form seems to be not very important,
we can learn something from the air-bag theory

Need to clarify the issue of the wake-field module
for long bunches, esp. low-freq. impedances

Clarify the role of space charge for PS/SIS100 head-tail modes

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Further studies for bunch stability with octupoles and SC

Suggestion: experiments for weak head-tail with octupoles at PS