**PROCEDURES FOR FREQUENCY AND TIME DOMAIN ELECTROMAGNETIC SIMULATIONS IN ASYMMETRIC STRUCTURES**

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**Reminder on the issue:**
- There is no problem for the longitudinal plane.
- The problem lies in the transverse planes. For more precise analyses of machine impedances and related collective effects, several quantities are needed:
  - Driving (or dipolar) impedance in the horizontal plane.
  - Driving (or dipolar) impedance in the vertical plane.
  - Detuning (or quadrupolar) impedance.
- Usually, depending on how the simulations (or measurements) are made, either only the driving impedance is obtained, or the sum (or difference) of the driving and detuning impedances.
- The purpose of this paper is to try to set up general procedures to obtain all the relevant quantities from both frequency and time domain analyses.

1) **FREQUENCY DOMAIN => Valid for both simulations (e.g. with HFSS) and measurements (using wires)**

- The “general” transverse impedances \( Z_{x,y} \) (in \( \Omega \), i.e. not normalized by the transverse displacement) on a test particle at \((x_2 = a_2 \cos \theta_2, y_2 = a_2 \sin \theta_2)\) from a source at \((x_1 = a_1 \cos \theta_1, y_1 = a_1 \sin \theta_1)\), are given by (to 1st order):

\[
\begin{align*}
Z_x[\Omega] & = \frac{1}{k} \left\{ \left( Z_{0,1} + Z_{0,-1} \right) + x_1 \bar{Z}_x + j y_1 \left( -Z_{1,-1} - Z_{1,1} + Z_{-1,-1} + Z_{-1,1} \right) \right. \\
& \quad + 2 \left( Z_{0,2} + Z_{0,-2} \right) x_2 + 2 \left( Z_{0,2} - Z_{0,-2} \right) j y_2 \\
& \quad + \left. \left( -2 \left( Z_{0,2} + Z_{0,-2} \right) y_2 + 2 \left( Z_{0,2} - Z_{0,-2} \right) j x_2 \right) \right\},
\end{align*}
\]

(1)

where \( Z_{m,n} \) are some coefficients, and with

\[
k = \frac{\omega}{c}, \quad \bar{Z}_x = Z_{1,1} + Z_{-1,-1} + Z_{1,-1} + Z_{-1,1}, \quad \bar{Z}_y = Z_{1,1} - Z_{1,-1} - Z_{-1,1} + Z_{-1,-1}.
\]

(2)

- By definition, the horizontal and vertical driving impedances are given by

\[
Z_{x,y}^{\text{driving}} \left[ \Omega / m \right] = \bar{Z}_{x,y} / k,
\]

(3)

while the detuning impedance is given by

\[
Z^{\text{detuning}} \left[ \Omega / m \right] = -2 \left( Z_{0,2} + Z_{0,-2} \right) / k.
\]

(4)

- Neglecting the constants and coupling terms, Eq. (1) can be re-written

\[
Z_x[\Omega] = x_1 Z_x^{\text{driving}} - Z^{\text{detuning}} x_2,
\]
\[ Z_y \left[ \Omega \right] = y_1 Z_y^{\text{driving}} + Z_y^{\text{detuning}} y_2. \]

- It can be clearly seen from Eq. (5) that:

1) In axi-symmetric structures, \( Z_x^{\text{driving}} = Z_y^{\text{driving}} \) and \( Z_y^{\text{detuning}} = 0 \).

2) If \( x_2 = x_1 \) and \( y_2 = y_1 \), then the transverse impedances which are obtained are not the driving ones, but the “generalized” ones

\[
Z_x^{\text{generalized}} \left[ \Omega / \text{m} \right] = \frac{Z_x}{x_1} = Z_x^{\text{driving}} - Z_x^{\text{detuning}},
\]

\[ Z_y^{\text{generalized}} \left[ \Omega / \text{m} \right] = \frac{Z_y}{y_1} = Z_y^{\text{driving}} + Z_y^{\text{detuning}}. \]

- Usually the longitudinal impedance \( Z \) is obtained through simulations (or measurements) using 1 wire (simulating the beam) and the transverse impedance is obtained with 2 wires (spaced by \( \pm a \), with opposite current, simulating a dipole). With the 2-wire method, only the driving (or dipolar) impedance is obtained. The procedure is the following: simulate (or measure) the longitudinal impedance \( Z \) and deduce the transverse one

\[
Z = 2 Z_{ch} \frac{1 - S_{21}}{S_{21}} , \quad Z_x^{\text{driving}} = \frac{Z_x}{k} = \frac{c Z}{\omega \left( 2 a \right)^2},
\]

where \( Z_{ch} \) is the characteristic impedance and \( S_{21} \) the scattering parameter. The same thing has to be done in the vertical plane to obtain \( Z_y^{\text{driving}} \).

- The transverse generalized impedances can be obtained using 1 wire (at \( x = a \cos \theta \), \( y = a \sin \theta \)). The longitudinal impedance simulated (or measured) is given by (to 2nd order)

\[
Z = A_1 + a e^{-j \theta} A_2 + a e^{j \theta} A_3 + a^2 e^{-2j \theta} A_4 + a^2 e^{2j \theta} A_5 + a^2 A_6,
\]

where \( A_1 = Z_{0,0}, \quad A_2 = Z_{1,0} + Z_{0,-1}, \quad A_3 = Z_{0,0} + Z_{0,-1}, \quad A_4 = Z_{2,0} + Z_{1,-1} + Z_{0,-2}, \quad A_5 = Z_{0,2} + Z_{1,1} + Z_{2,0} \) and \( A_6 = Z_{1,1} + Z_{1,-1} \).

1) If there is top/bottom and left/right symmetry (fortunately it is the usual case...), the situation simplifies a lot. See EPAC06 paper (http://accelconf.web.cern.ch/AccelConf/e06/PAPERS/THPCH059.PDF):

- If \( a = x_0 \) and \( \theta = 0 \):

\[
Z = A_1 + x_0^2 (A_1 + A_2 + A_6) = A_1 + x_0^2 \left[ Z_x + \left( Z_{2,0} + Z_{2,0} + Z_{0,2} + Z_{0,2} \right) \right].
\]

Scanning \( x_0 \) gives a parabola. The impedance which can then be obtained is

\[
Z_{l,1x} = Z_x + \left( Z_{2,0} + Z_{0,2} + Z_{0,-2} + Z_{0,-2} \right). \]
- Similarly, if \( a = y_0 \) and \( \theta = \pi/2 \):

\[
Z = A_1 + y_0^2 \left( -A_4 - A_5 + A_6 \right) = A_1 + y_0^2 \left[ Z_y - \left( Z_{2,0} + Z_{2,0} + Z_{-2,0} + Z_{0,-2} \right) \right].
\] (11)

Scanning \( y_0 \) gives also a parabola. The impedance which can then be obtained is

\[
Z_{l,1,y} = Z_y - \left( Z_{2,0} + Z_{2,0} + Z_{-2,0} + Z_{0,-2} \right).
\] (12)

- Now, if the following relation is satisfied (still to be demonstrated in the general case \( \Rightarrow \) Bruno Zotter?)

\[
Z_{2,0} + Z_{-2,0} = Z_{0,2} + Z_{0,-2},
\] (13)

then

\[
Z_{2,0} + Z_{0,2} + Z_{-2,0} + Z_{0,-2} = 2 \left( Z_{0,2} + Z_{0,-2} \right) = -k Z^\text{detuning}.
\] (14)

Therefore, in this case,

\[
Z^\text{generalized}_{x} \left[ \Omega / m \right] = Z^\text{driving}_{x} - Z^\text{detuning} = \frac{Z_{l,1,x}}{k},
\] (15)

\[
Z^\text{generalized}_{y} \left[ \Omega / m \right] = Z^\text{driving}_{y} + Z^\text{detuning} = \frac{Z_{l,1,y}}{k}.
\]

- In conclusion, the driving impedances are obtained from 2-wire measurements and the generalized impedances are obtained through 1-wire measurements. Therefore, the detuning impedance can be deduced. A nice cross-check to do is to sum the 2 driving impedances obtained through 2-wire measurements and verify that it is equal to the sum of the 2 generalized impedances (as the detuning impedance disappears!).

- **2) If there is NO top/bottom or left/right symmetry, the situation is more involved:**

- By scanning \( a \) and \( \theta \) (i.e. measuring \( Z \) for different values of \( a \) and \( \theta \)), \( A_{1,2,3,4,5,6} \) can be found.

- Using the 2-wire technique the driving impedances \( Z^\text{driving}_{x,y} \) can be obtained (as before).

- Then, if Eq. (13) is satisfied (which still has to be demonstrated in the general case),

\[
Z^\text{detuning} = \frac{Z^\text{driving}_{x} - Z^\text{driving}_{y}}{2} = \frac{A_4 + A_5}{k}.
\] (16)

- In conclusion, the driving impedances are obtained from 2-wire measurements and the detuning impedance can be deduced from 1-wire measurements.
2) TIME DOMAIN => Valid for simulations

- Neglecting the constants and coupling terms, Eq. (5) can be re-written in time-domain as follows:

\[
\int_L ds \, F_x = -q^2 \left[ x_1 W_x^{\text{driving}}(z) - x_2 W_x^{\text{detuning}}(z) \right],
\]

(17)

\[
\int_L ds \, F_y = -q^2 \left[ y_1 W_y^{\text{driving}}(z) + y_2 W_y^{\text{detuning}}(z) \right].
\]

- Note that \( \int ds \, F_{x,y} \) are called the transverse wake potentials, whereas \( W_{x,y}^{\text{driving}}(z) \) are the (driving) wake functions and \( W_{x,y}^{\text{detuning}}(z) \) the (detuning) wake function, where \( z \) is the distance between the leading (source) and trailing (test) particle.

- In this case, the idea is to make 2 simulations:

- The first with \( x_1 \neq 0 \) and looking at the transverse field map vs. \( z \) for \( x_2 = x_1 \), and similarly in the vertical plane. This gives the contribution of the “generalized” wake functions

\[
W_x^{\text{generalized}}(z) = W_x^{\text{driving}}(z) - W_x^{\text{detuning}}(z) = -\frac{1}{q^2} x_1 \int_L ds \, F_x,
\]

(18)

\[
W_y^{\text{generalized}}(z) = W_y^{\text{driving}}(z) + W_y^{\text{detuning}}(z) = -\frac{1}{q^2} y_1 \int_L ds \, F_y.
\]

- The second with \( x_1 = 0 \) and looking at the transverse field map vs. \( z \) for \( x_2 \neq 0 \). In this case the detuning wake function is obtained

\[
W_x^{\text{detuning}}(z) = -\frac{1}{q^2} x_2 \int_L ds \, F_x.
\]

(19)

- A good check to make is to try and obtain the detuning wake function from the vertical plane with \( y_1 = 0 \) and looking at the transverse field map vs. \( z \) for \( y_2 \neq 0 \),

\[
W_y^{\text{detuning}}(z) = -\frac{1}{q^2} y_2 \int_L ds \, F_y,
\]

(20)

which should give the same result as the one obtained from Eq. (19).

- From Eqs. (18) and (19) (or Eq. (20)), the horizontal and vertical wake functions can be obtained in addition to the detuning one.