PROCEDURES FOR FREQUENCY AND TIME DOMAIN ELECTRO-MAGNETIC SIMULATIONS IN ASYMMETRIC STRUCTURES

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Reminder on the issue:

- There is no problem for the longitudinal plane.
- The problem lies in the transverse planes. For more precise analyses of machine impedances and related collective effects, several quantities are needed:
    - Driving (or dipolar) impedance in the horizontal plane.
    - Driving (or dipolar) impedance in the vertical plane.
    - Detuning (or quadrupolar) impedance.
- Usually, depending on how the simulations (or measurements) are made, either only the driving impedance is obtained, or the sum (or difference) of the driving and detuning impedances.
- The purpose of this paper is to try to set up general procedures to obtain all the relevant quantities from both frequency and time domain analyses.

1) FREQUENCY DOMAIN => Valid for both simulations (e.g. with HFSS) and measurements (using wires)

- The "general" transverse impedances $Z_{x,y}$ (in Ω!, i.e. not normalized by the transverse displacement) on a test particle at ($x_2 = a_2 \cos\theta_2$, $y_2 = a_2 \sin\theta_2$) from a source at ($x_1 = a_1 \cos\theta_1$, $y_1 = a_1 \sin\theta_1$), are given by (to 1st order):

$$Z_x\left[\Omega\right] = \frac{1}{k}\left\{\begin{array}{c}\left(Z_{0,1} + Z_{0,-1}\right) + x_1\,\overline{Z}_x + j\,y_1\left(-Z_{1,-1} - Z_{1,1} + Z_{-1,-1} + Z_{-1,1}\right)\\ + 2\left(Z_{0,2} + Z_{0,-2}\right)x_2 + 2\left(Z_{0,2} - Z_{0,-2}\right)j\,y_2\end{array}\right\},$$

$$Z_y\left[\Omega\right] = \frac{1}{k}\left\{\begin{array}{c}j\left(Z_{0,1} - Z_{0,-1}\right) + y_1\,\overline{Z}_y + j\,x_1\left(-Z_{1,-1} + Z_{1,1} - Z_{-1,-1} + Z_{-1,1}\right)\\ - 2\left(Z_{0,2} + Z_{0,-2}\right)y_2 + 2\left(Z_{0,2} - Z_{0,-2}\right)j\,x_2\end{array}\right\}, \qquad (1)$$

where $Z_{m,n}$ are some coefficients, and with

$$k = \omega / c\,, \qquad \overline{Z}_x = Z_{1,1} + Z_{1,-1} + Z_{-1,1} + Z_{-1,-1}\,, \qquad \overline{Z}_y = Z_{1,1} - Z_{1,-1} - Z_{-1,1} + Z_{-1,-1}\,. \qquad (2)$$

- By definition, the horizontal and vertical driving impedances are given by

$$Z_{x,y}^{\text{driving}}\left[\Omega/\text{m}\right] = \overline{Z}_{x,y} \,/\, k\,, \qquad (3)$$

while the detuning impedance is given by

$$Z^{\text{detuning}}\left[\Omega/\text{m}\right] = -2\left(Z_{0,2} + Z_{0,-2}\right)/\,k\,. \qquad (4)$$

- Neglecting the constants and coupling terms, Eq. (1) can be re-written

$$Z_x\left[\Omega\right] = x_1\,Z_x^{\text{driving}} - Z^{\text{detuning}}\;x_2\,,$$

(5)

$$Z_y\left[\Omega\right] = y_1\, Z_y^{\text{driving}} + Z^{\text{detuning}}\, y_2\,.$$

- It can be clearly seen from Eq. (5) that:

1) In axi-symmetric structures, $Z_x^{\text{driving}} = Z_y^{\text{driving}}$ and $Z^{\text{detuning}} = 0$.

2) If $x_2 = x_1$ and $y_2 = y_1$, then the transverse impedances which are obtained are not the driving ones, but the "generalized" ones

$$Z_x^{generalized}\left[\Omega / \text{m}\right] = \frac{Z_x}{x_1} = Z_x^{\text{driving}} - Z^{\text{detuning}}\,,$$

(6)

$$Z_y^{generalized}\left[\Omega / \text{m}\right] = \frac{Z_y}{y_1} = Z_y^{\text{driving}} + Z^{\text{detuning}}\,.$$

- Usually the longitudinal impedance Z is obtained through simulations (or measurements) using 1 wire (simulating the beam) and the transverse impedance is obtained with 2 wires (spaced by $\pm a$, with opposite current, simulating a dipole). With the 2-wire method, only the driving (or dipolar) impedance is obtained. The procedure is the following: simulate (or measure) the longitudinal impedance Z and deduce the transverse one

$$Z = 2\, Z_{ch} \frac{1 - S_{21}}{S_{21}}\,, \qquad Z_x^{\text{driving}} = \frac{\overline{Z}_x}{k} = \frac{c\, Z}{\omega \left(2\, a\right)^2}\,, \tag{7}$$

where Z_{ch} is the characteristic impedance and S_{21} the scattering parameter. The same thing has to be done in the vertical plane to obtain Z_y^{driving}.

- The transverse generalized impedances can be obtained using 1 wire (at x = a cosθ, y = a sinθ). The longitudinal impedance simulated (or measured) is given by (to 2nd order)

$$Z = A_1 + a\, e^{-j\vartheta} A_2 + a\, e^{j\vartheta} A_3 + a^2\, e^{-2j\vartheta} A_4 + a^2\, e^{2j\vartheta} A_5 + a^2 A_6\,, \tag{8}$$

where $A_1 = Z_{0,0}$, $A_2 = Z_{1,0} + Z_{0,-1}$, $A_2 = Z_{1,0} + Z_{0,-1}$, $A_4 = Z_{2,0} + Z_{1,-1} + Z_{0,-2}$, $A_5 = Z_{0,2} + Z_{-1,1} + Z_{-2,0}$ and $A_6 = Z_{1,1} + Z_{-1,-1}$.

1) If there is top/bottom and left/right symmetry (fortunately it is the usual case…), the situation simplifies a lot $\Longrightarrow$ See EPAC06 paper (http://accelconf.web.cern.ch/AccelConf/e06/PAPERS/THPCH059.PDF):

- If a = x_0 and θ = 0:

$$Z = A_1 + x_0^2\left(A_4 + A_5 + A_6\right) = A_1 + x_0^2\left[\overline{Z}_x + \left(Z_{2,0} + Z_{0,2} + Z_{-2,0} + Z_{0,-2}\right)\right]. \tag{9}$$

Scanning x_0 gives a parabola. The impedance which can then be obtained is

$$Z_{l,1x} = \overline{Z}_x + \left(Z_{2,0} + Z_{0,2} + Z_{-2,0} + Z_{0,-2}\right). \tag{10}$$

- Similarly, if a = y_0 and $\theta = \pi/2$:

$$Z = A_1 + y_0^2 \left(- A_4 - A_5 + A_6 \right) = A_1 + y_0^2 \left[\overline{Z}_y - \left(Z_{2,0} + Z_{0,2} + Z_{-2,0} + Z_{0,-2} \right) \right] . \tag{11}$$

Scanning y_0 gives also a parabola. The impedance which can then be obtained is

$$Z_{l,1y} = \overline{Z}_y - \left(Z_{2,0} + Z_{0,2} + Z_{-2,0} + Z_{0,-2} \right). \tag{12}$$

- Now, if the following relation is satisfied (still to be demonstrated in the general case $\Longrightarrow$ Bruno Zotter?)

$$Z_{2,0} + Z_{-2,0} = Z_{0,2} + Z_{0,-2} , \tag{13}$$

then

$$Z_{2,0} + Z_{0,2} + Z_{-2,0} + Z_{0,-2} = 2 \left(Z_{0,2} + Z_{0,-2} \right) = - k \, Z^{\text{detuning}} . \tag{14}$$

Therefore, in this case,

$$Z_x^{generalized} \left[\Omega / \text{m} \right] = Z_x^{\text{driving}} - Z^{\text{detuning}} = \frac{Z_{l,1x}}{k} ,$$

$$(15)$$

$$Z_y^{generalized} \left[\Omega / \text{m} \right] = Z_y^{\text{driving}} + Z^{\text{detuning}} = \frac{Z_{l,1y}}{k} .$$

- In conclusion, the driving impedances are obtained from 2-wire measurements and the generalized impedances are obtained through 1-wire measurements. Therefore, the detuning impedance can be deduced. A nice cross-check to do is to sum the 2 driving impedances obtained through 2-wire measurements and verify that it is equal to the sum of the 2 generalized impedances (as the detuning impedance disappears!).

- 2) If there is NO top/bottom or left/right symmetry, the situation is more involved:

- By scanning a and θ (i.e. measuring Z for different values of a and θ), $A_{1,2,3,4,5,6}$ can be found.

- Using the 2-wire technique the driving impedances $Z_{x,y}^{\text{driving}}$ can be obtained (as before).

- Then, if Eq. (13) is satisfied (which still has to be demonstrated in the general case),

$$Z^{\text{detuning}} = \frac{Z_x^{\text{driving}} - Z_y^{\text{driving}}}{2} - \frac{A_4 + A_5}{k} . \tag{16}$$

- In conclusion, the driving impedances are obtained from 2-wire measurements and the detuning impedance can be deduced from 1-wire measurements.

2) TIME DOMAIN => Valid for simulations

- Neglecting the constants and coupling terms, Eq. (5) can be re-written in time-domain as follows

$$\int_L ds\, F_x = -q^2 \left[x_1\, W_x^{\text{driving}}(z) - x_2\, W^{\text{detuning}}(z) \right],$$

$$\int_L ds\, F_y = -q^2 \left[y_1\, W_y^{\text{driving}}(z) + y_2\, W^{\text{detuning}}(z) \right]. \tag{17}$$

- Note that $\int_L ds\, F_{x,y}$ are called the transverse wake potentials, whereas $W_{x,y}^{\text{driving}}(z)$ are the (driving) wake functions and $W^{\text{detuning}}(z)$ the (detuning) wake function, where z is the distance between the leading (source) and trailing (test) particle.

- In this case, the idea is to make 2 simulations:

- The first with $x_1 \neq 0$ and looking at the transverse field map vs. z for $x_2 = x_1$, and similarly in the vertical plane. This gives the contribution of the "generalized" wake functions

$$W_x^{generalized}(z) = W_x^{\text{driving}}(z) - W^{\text{detuning}}(z) = -\frac{1}{q^2\, x_1} \int_L ds\, F_x\,,$$

$$W_y^{generalized}(z) = W_y^{\text{driving}}(z) + W^{\text{detuning}}(z) = -\frac{1}{q^2\, y_1} \int_L ds\, F_y\,. \tag{18}$$

- The second with $x_1 = 0$ and looking at the transverse field map vs. z for $x_2 \neq 0$. In this case the detuning wake function is obtained

$$W^{\text{detuning}}(z) = -\frac{1}{q^2\, x_2} \int_L ds\, F_x\,. \tag{19}$$

- A good check to make is to try and obtain the detuning wake function from the vertical plane with $y_1 = 0$ and looking at the transverse field map vs. z for $y_2 \neq 0$,

$$W^{\text{detuning}}(z) = -\frac{1}{q^2\, y_2} \int_L ds\, F_y\,, \tag{20}$$

which should give the same result as the one obtained from Eq. (19).

- From Eqs. (18) and (19) (or Eq. (20)), the horizontal and vertical wake functions can be obtained in addition to the detuning one.