1- and 2-wire transverse impedance measurement or simulation

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See http://www.slac.stanford.edu/cgi-wrap/getdoc/slac-ap-110.pdf (Heifets et al.) and http://cdsweb.cern.ch/record/702715/files/sl-note-2002-034.pdf (Tsutsui)

- **For more precise analyses of machine impedances and related collective effects, several quantities are needed**
  - $Z_{x,dip}$ **(dipolar or driving)**
  - $Z_{x,quad}$ **("quadrupolar" or detuning)**
  - $Z_{y,dip}$ **(driving)**
  - $Z_{y,quad}$ **(detuning)**

Z_{quad} is the same for both transverse planes $\Longrightarrow$ BUT, 2 differences for its use: 1) Subtracts in x and adds in y; 2) Impedances are weighted by the betatron function in each plane

“General” definition of transverse impedances (1/2)

- **Axi-symmetric structures ⟹ A current density with some azimuthal Fourier component creates electromagnetic fields with the same azimuthal Fourier component**

$$\overline{Z}_m = -\frac{1}{Q^2}\int dV\, \overline{E}_m \,.\, \overline{J}_m^{\,*}$$

Usual definition of the longitudinal impedance (m=0,1,2,…)

- **Non axi-symmetric structures ⟹ A current density with some azimuthal Fourier component may create an electromagnetic field with various different azimuthal Fourier components ⟹ A more general beam coupling impedance is defined in order to treat coupling of different azimuthal Fourier components**

$$Z_{m,n} = -\frac{1}{Q^2}\int dV\, E_m \,.\, J_n^{\,*}$$

More “general” definition of the longitudinal impedance (m,n = 0, ±1, ±2,…)

"General" definition of transverse impedances (2/2)

- **The "general" transverse impedances $Z_{x,y}$ (in Ω!, i.e. not normalized by the transverse displacement) on a test particle at ($x_2 = a_2 \cos\theta_2$, $y_2 = a_2 \sin\theta_2$) from a source at ($x_1 = a_1 \cos\theta_1$, $y_1 = a_1 \sin\theta_1$), are given by (to 1st order)**

$$k\, Z_x = \left(Z_{0,1} + Z_{0,-1} \right) + x_1\, \overline{Z}_x + j\, y_1 \left(- Z_{1,-1} - Z_{1,1} + Z_{-1,-1} + Z_{-1,1} \right) + 2 \left(Z_{0,2} + Z_{0,-2} \right) x_2 + 2 \left(Z_{0,2} - Z_{0,-2} \right) j\, y_2$$

$$k\, Z_y = j \left(Z_{0,1} - Z_{0,-1} \right) + y_1\, \overline{Z}_y + j\, x_1 \left(- Z_{1,-1} + Z_{1,1} - Z_{-1,-1} + Z_{-1,1} \right) - 2 \left(Z_{0,2} + Z_{0,-2} \right) y_2 + 2 \left(Z_{0,2} - Z_{0,-2} \right) j\, x_2$$

with $k = \omega / c$ $\quad \overline{Z}_x = Z_{1,1} + Z_{1,-1} + Z_{-1,1} + Z_{-1,-1}$ $\quad \overline{Z}_y = Z_{1,1} - Z_{1,-1} - Z_{-1,1} + Z_{-1,-1}$

$\Longrightarrow$ $\quad Z_x^{\text{driving}} = \overline{Z}_x / k$ $\quad Z_y^{\text{driving}} = \overline{Z}_y / k$ $\quad Z^{\text{detuning}} = -2 \left(Z_{0,2} + Z_{0,-2} \right) / k$

2-wire technique

- **This gives the (usual) dipolar (driving) impedance**
- **Method: Measure the longitudinal impedance Z and deduce the transverse one**

$$Z = 2\, Z_{ch} \, \frac{1 - S_{21}}{S_{21}}$$

$$Z_x^{\text{driving}} = \frac{\overline{Z}_x}{k} = \frac{c\, Z}{\omega \left(2\, a \right)^2}$$

Characteristic impedance

Scattering parameter

The 2 wires are at ± a

1-wire technique (1/4)

- **With 1 wire (at x = a cosθ, y = a sinθ), the longitudinal impedance measured (or simulated) is given by (to 2nd order)**

$$Z = A_1 + a\, e^{-j\vartheta} A_2 + a\, e^{j\vartheta} A_3 + a^2 e^{-2j\vartheta} A_4 + a^2 e^{2j\vartheta} A_5 + a^2 A_6$$

with

$$A_1 = Z_{0,0}$$

$$A_2 = Z_{1,0} + Z_{0,-1}$$

$$A_3 = Z_{0,1} + Z_{-1,0}$$

$$A_4 = Z_{2,0} + Z_{1,-1} + Z_{0,-2}$$

$$A_5 = Z_{0,2} + Z_{-1,1} + Z_{-2,0}$$

$$A_6 = Z_{1,1} + Z_{-1,-1}$$

1-wire technique (2/4)

- **If there is top/bottom and left/right symmetry (fortunately it is the usual case…), the situation simplifies a lot ⟹ See EPAC06 paper (http://accelconf.web.cern.ch/AccelConf/e06/PAPERS/THPCH059.PDF)**

  - **If a = x_0 and $\theta = 0$**

$$Z = A_1 + x_0^2 \left(A_4 + A_5 + A_6 \right)$$
$$= A_1 + x_0^2 \left[\overline{Z}_x + \left(Z_{2,0} + Z_{0,2} + Z_{-2,0} + Z_{0,-2} \right) \right]$$

Scanning x_0 gives a parabola

  - **If a = y_0 and $\theta = \pi/2$**

$$Z = A_1 + y_0^2 \left(- A_4 - A_5 + A_6 \right)$$
$$= A_1 + y_0^2 \left[\overline{Z}_y - \left(Z_{2,0} + Z_{0,2} + Z_{-2,0} + Z_{0,-2} \right) \right]$$

1-wire technique (3/4)

$\Longrightarrow$ **IF** $Z_{2,0} + Z_{-2,0} = Z_{0,2} + Z_{0,-2}$ **, then**

$$Z_{2,0} + Z_{0,2} + Z_{-2,0} + Z_{0,-2} = 2\left(Z_{0,2} + Z_{0,-2}\right) = -\,k\,Z^{\text{detuning}}$$

Tsutsui showed "approximately" that for a 2D thick metallic boundary case: $Z_{-m,-n} = Z_{n,m}$ $\Longrightarrow$ In this case, the above formula is indeed valid

Remark 1: Only the "generalized impedances" can be obtained but not the driving and detuning impedance separately! This cannot be used like this in codes like HEADTAIL for instance

Remark 2: The dipolar (driving) impedances need to be obtained from the 2-wire technique

1-wire technique (4/4)

- **If there is NO top/bottom or left/right symmetry, the situation is more involved:**
  - **By scanning a and θ (i.e. measuring Z for different values of a and θ), $A_{1,2,3,4,5,6}$ can be found**
  - **Then, using the 2-wire technique the dipolar (driving) impedances can be obtained:** $Z_x^{\text{driving}} = \overline{Z}_x / k$ $\quad Z_y^{\text{driving}} = \overline{Z}_y / k$
  - **Then compute** $Z_{1,-1} + Z_{-1,1} = \left(\overline{Z}_x - \overline{Z}_y \right) / 2$
  - **Then, if** $Z_{2,0} + Z_{-2,0} = Z_{0,2} + Z_{0,-2}$ **,**

$$Z^{\text{detuning}} = \frac{Z_x^{\text{driving}} - Z_y^{\text{driving}}}{2} - \frac{A_4 + A_5}{k}$$

Conclusion (and reminder)

- Both 1-wire and 2-wire techniques are required (in asymmetric structures) to obtain all the information needed to correctly understand/describe the collective effects in accelerators

- With 2 wires the transverse dipolar (driving) impedances are obtained

- With 1 wire (scanning the wire position), and using the driving impedances measured with 2 wires, the detuning impedance can be deduced (IF a certain condition is fulfilled $\Longrightarrow$ Still to be checked in which cases this relation is satisfied or not)

- What about the coupling terms (see page 3)??? Can they be important in some cases???